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An Approach to Calculating Live Load Distribution Factors for Horizontally Curved Bridges with Straight Girders Using Finite Element Analysis

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Abstract:

Live load distribution factors (LLDF)s of beam-slab bridges of select typical configurations can be calculated using the approximate equations given by AASHTO BDS 2021. However, these equations are applicable primarily for straight bridges with limited exceptions for curved ones. It is common to use a beam-slab construction with straight prestressed concrete girders in horizontally curved bridges in the United States. For such cases, finite element analysis must be used to calculate LLDFs.

This study developed an approach to calculating LLDFs using generalized finite element tools including geometrical simplifications in models, application of vehicular live load, and data analysis required to calculate the LLDFs. To present the efficiency of the developed approach, one straight bridge and one curved bridge with a moderate degree of horizontal curvature were modeled and analyzed to investigate the effects of bridge curvature. Using the set of simplifications detailed in the approach, secondary structural or non-structural components of the bridges (parapets, traffic features, etc.) were not modeled. The results showed the developed approach could well consider the complex geometry of bridges and the effects of curvatures, span length, and the number of traffic lanes. Further research regarding the curvature effects on LLDF in curved bridges was also discussed.

1. Introduction

In the United States, the transfer of vehicular live load from the deck to the girders of a beam-slab bridge is calculated indirectly through live load distribution factors (LLDF)s. The American Association of State Highway and Transportation Officials (AASHTO) provides guidelines on calculating LLDFs in the AASHTO Load and Resistance Factored Design (LRFD) Bridge Design Specifications (BDS) Article 4.6 (AASHTO, 2020). AASHTO BDS provides both approximate equations to directly calculate LLDFs and guidelines for refined analysis methods. The approximate equations have been developed over the course of 90 years from research conducted using a refined analysis and mathematical modeling. While these equations help to simplify the preliminary design process, their applicability to horizontally curved bridges is limited (AASHTO, 2020). For curved bridges with eccentric girder geometry, refined analysis must be used to calculate the load transfer to the girders. In the modern design process, finite element analysis is favored heavily by engineers over more archaic hand calculation methods and is used almost exclusively.

1.1 AASHTO BDS Provisions for Vehicular Live Load Application

The procedures for calculating the vehicular live load on a bridge deck, referred to as the bridge roadway in AASHTO BDS, are outlined in Article 3.6 of the specifications. Per these specifications, a design truck (HL-93) is applied over one or more design lanes to find the maximum vehicular live load that the bridge may experience, referred to as the extreme force effect (AASHTO, 2020). The number of design lanes for a bridge is dependent on the width of the bridge roadway and can be determined using the provisions in AASHTO BDS in Article 3.6.1.1.1. The dimensions and forces at the axles of the HL-93 design truck is presented in Figure 1.

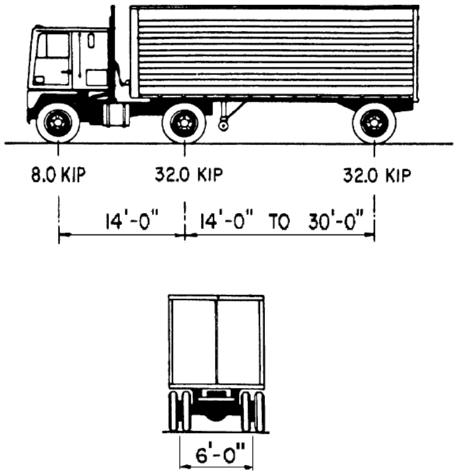


Figure 1: AASHTO HL-93 Design Truck and Design Tandem (AASHTO, 2020)

Per AASHTO BDS Article 3.6.1.1.2, the extreme live load force effect may be calculated by applying the HL-93 design load over every combination of the design lanes using the multiple presence factors presented in the specifications (AASHTO, 2020). AAHSTO BDS Article 3.6.1.3 presents additional information about calculating the extreme force effect and requirements for applying the vehicular live load.

1.2 AASHTO BDS Provisions for Finite Element Analysis

AASHTO BDS provides general requirements and recommendations for finite element modeling of bridges in Article 4.6.3. AASHTO BDS Article 4.6.3.2 defines guidelines for the finite element modeling of a bridge deck. For deformation analysis, flexural and torsional deformation shall be considered while vertical shear deformation may be neglected (AASHTO, 2020). For modeling an orthotropic deck slab, three-dimensional finite shell elements or solid elements are recommended, and all components shall be included in the model. AASHTO Article 4.6.3.3 provides guidelines for the aspect ratio of elements when modeling a beam-slab bridge. AASHTO requires that the ratio of finite elements/grid panels not exceed 5.0 and recommends against abrupt changes in element geometry.

1.3 Significance and Objectives of This Study

This study focused on finite element analysis of horizontally curved bridge superstructures supported by straight prestressed concrete girders. In the United States, construction of curved bridge superstructures with straight underlying precast concrete I-girders is preferred over curved precast concrete I-girders due to complexity in fabrication and transportation (Amorn et al., 2008). As previously stated, the eccentric geometry of these types of bridges do not allow the use of the approximate LLDF equations by AASHTO BDS. Therefore, finite element analysis has to be used in all steps of the design to calculate the load transfers from the deck to the girders.

The objective of this study was to, using criteria derived from AASHTO and from literature (Chen & Aswad, 1996; Zaki, 2016), develop a set of guidelines on geometrical simplifications for engineers to quickly model bridges with complex geometry, and to investigate the effects of bridge curvature on LLDFs. These guidelines were used to model and analyze one straight bridge and one bridge with a moderate degree of horizontal curvature. From the results of the finite element analyses, the LLDF to each girder at each span was calculated and the variations of these factors were discussed. Conclusions were drawn from the results of this parametric study and recommendations for further research were made.

2 Simplifications for Finite Element Models

Simplifications for the finite element modeling of the horizontally curved bridges were developed using the provisions of AASHTO BDS. Additionally, the findings in literature to limit the factors affecting LLDFs were used to isolate and investigate the effect of bridge curvature on LLDFs. Geometrical simplifications were also developed in modeling bridges.

Based on the relevance to the approximate LLDF equations, a few bridge components were eliminated from the finite element models. The substructure components of the bridge (abutments, piers, and foundation) were eliminated from the models since these components do not directly influence the load distribution factors (Mannering & Washburn, 2013). The restraint mechanisms on the girders were included at the location of the piers. Concrete parapets and traffic structures on the bridge decks were removed from the models to simplify the load distribution from the concrete deck. Mid-span diaphragms were removed from the models since the reduction in deformation in a bridge superstructure provided by these diaphragms also provide a reduction in the load distribution factors (Chen & Aswad, 1996). In addition, vertical curvature and skew effects were neglected.

3 Finite Element Analysis

Finite element models were created and analyzed for the bridges using existing construction drawings in the structural software package SAP2000 (Version 19.0.0; CSI, 2016).

3.1 Description of Finite Element Models

The geometry of the curved bridge deck was determined using an arc fitting equation while the underlying straight girders with a linear slope equation. The calculations were performed at set discretization intervals in accordance with the acceptable aspect ratios presented by AASHTO BDS. Using spreadsheets, the coordinates were imported into SAP2000 as special nodes.

Using recommendations from literature, the element types for each bridge component was selected. To model the bridge deck, a series of four-node shell elements were used (Zaki, 2016). To model the girders, beam elements (referred to by SAP2000 as frame elements) were used to model the underlying girders. Additionally, rigid links with constrained vertical DOFs were used to simulate the connection between the deck and girder elements for load transfer (Zaki, 2016). For the structural loading of the AASHTO HL-93 design truck, a path along the bridge deck was created using frames with no assigned section properties. Since SAP2000 uses frame elements to create vehicle paths, null section frame elements can be used to effectively transfer vehicle loads to area elements that are meshed to the connecting nodes.

To efficiently generate bridge models, a numbering scheme was utilized for the nodes that allowed for frames, areas, links, and vehicle paths to be easily created and modified. The finite element models were created using SAP2000's database editor with external spreadsheet data with these pre-defined properties. Once the model was generated, any geometric or element errors could be easily diagnosed and corrected.

3.2 Modeled Bridges

Using the modeling scheme described above, one horizontally curved bridge with straight girders was modeled using SAP2000. The bridge that was selected is a four span, 410'-0"± long highway bridge located in Somerset County, Pennsylvania, USA with the coordinates N 39.82127°, W -79.04704° and a degree of curvature of 15°. To accurately model the bridge, detailed construction drawings provided by the Pennsylvania Department of Transportation were used (PennDOT, 1996). An elevation view of the bridge showing the span lengths and connections between the superstructure and substructure is presented in Figure 2. A framing plan showing the position of the supporting girders underneath the curved bridge deck is presented in Figure 3. A cross-section view showing the spacing of the girders and slope of the bridge deck is presented in Figure 4.

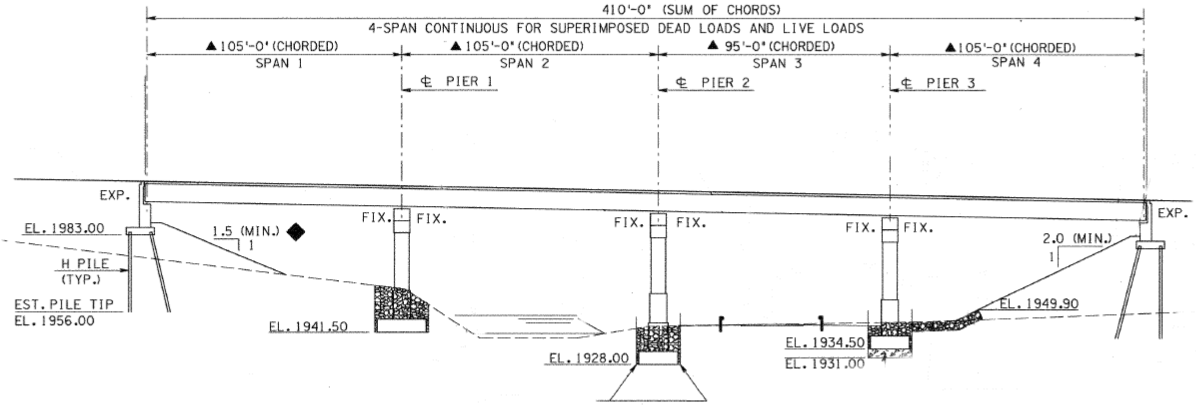


Figure 2: Elevation View of Horizontally Curved Bridge (PennDOT, 1996)

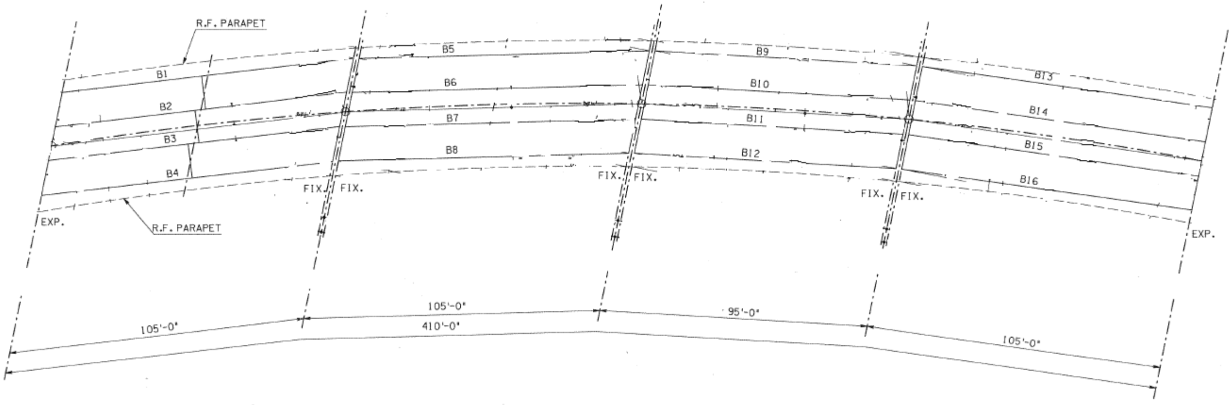


Figure 3: Framing Plan of Horizontally Curved Bridge (PennDOT, 1996)

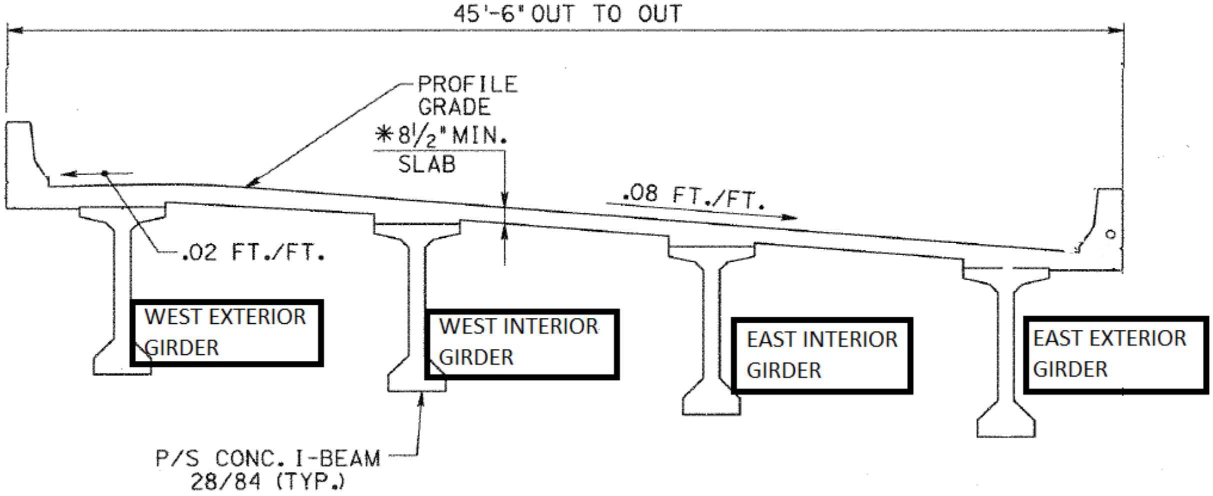
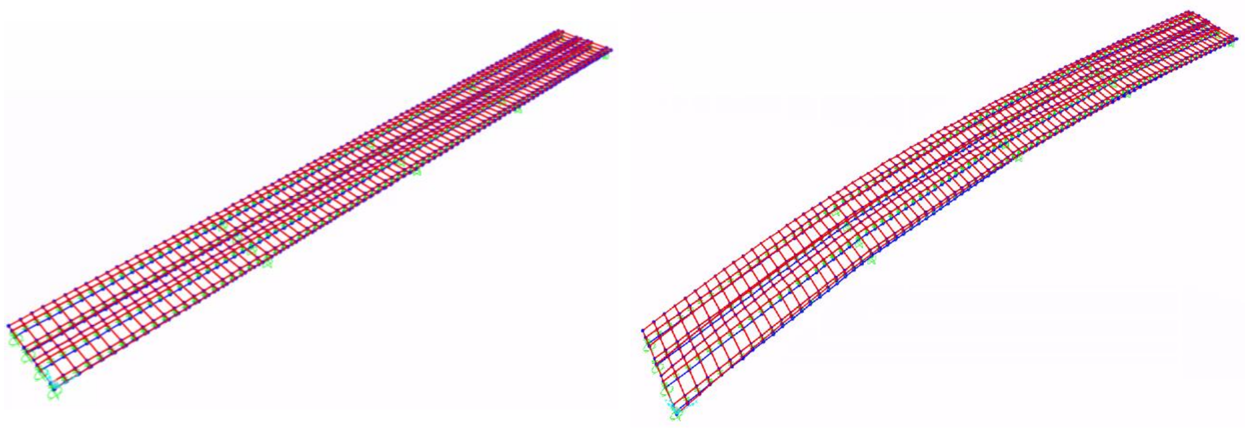


Figure 4: Cross-Section of Horizontally Curved Bridge (PennDOT, 1996)

To analyze the impact of curvature on LLDFs, a control straight bridge was created using the geometric properties and material properties of the horizontally curved bridge. This straight bridge used the same deck and girder material, cross-section geometry, number of spans, span length, and substructure connection types as the horizontally curved bridge. The finite element models of the straight bridge and the curved bridge are presented in Figure 5.



(a)

(b)

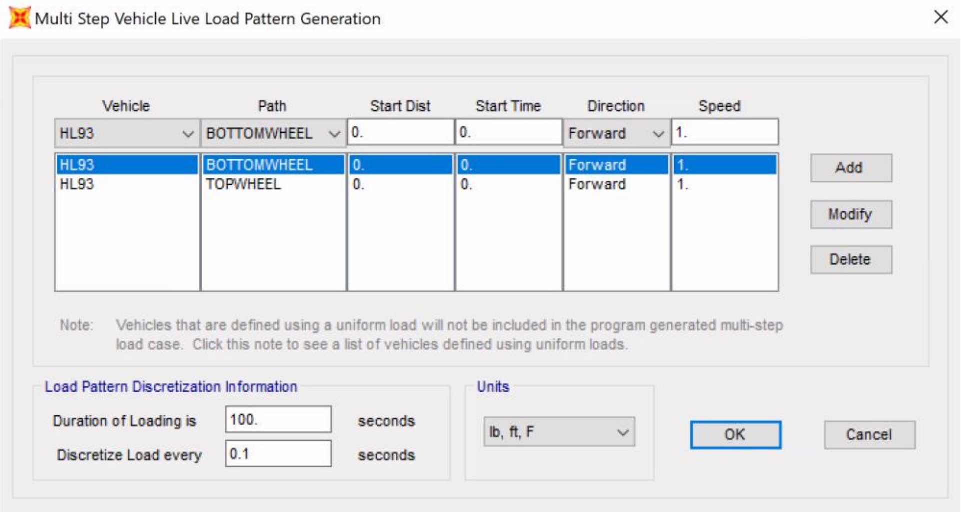
Figure 5: Finite Element Models: (a) Straight Bridge; and (b) Curved Bridge

3.3 Application of Live Load

Finite element analysis was conducted on the models presented in Figure 5. To demonstrate the methodology used to model and load the bridges, the bridge deck was analyzed in only one load case where the design truck was loaded at the center of the deck. To apply the design truck load in SAP2000, a

2D vehicle load was defined using half of the axle weight of the truck, as seen in Figure 1. Two lanes were modeled and loaded with this defined vehicle at 6’-0” apart to simulate the full axle weights of the truck.

In SAP2000, a moving load case and a vehicle live load pattern was defined to move the AASHTO HL-93 design truck along the designated path. The parameters of the vehicle live load pattern and moving load case are presented in Figure 6 and Figure 7 respectively.



Multi Step Vehicle Live Load Pattern Generation

Vehicle	Path	Start Dist	Start Time	Direction	Speed
HL93	BOTTOMWHEEL	0.	0.	Forward	1.
HL93	BOTTOMWHEEL	0.	0.	Forward	1.
HL93	TOPWHEEL	0.	0.	Forward	1.

Note: Vehicles that are defined using a uniform load will not be included in the program generated multi-step load case. Click this note to see a list of vehicles defined using uniform loads.

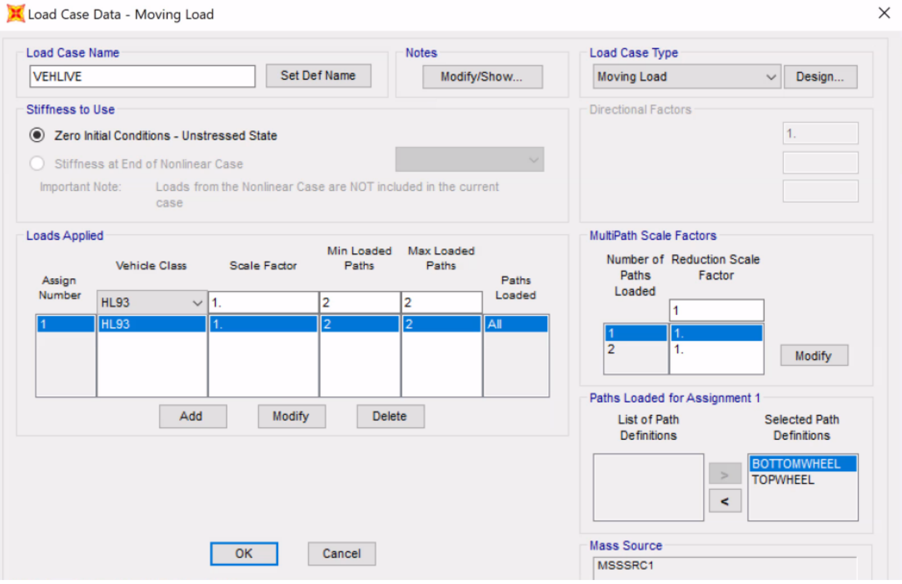
Load Pattern Discretization Information

Duration of Loading is 100. seconds

Discretize Load every 0.1 seconds

Units: lb, ft, F

Figure 6: Vehicle Live Load Pattern Parameters in SAP2000



Load Case Data - Moving Load

Load Case Name: VEHLIVE

Load Case Type: Moving Load

Stiffness to Use

● Zero Initial Conditions - Unstressed State

○ Stiffness at End of Nonlinear Case

Important Note: Loads from the Nonlinear Case are NOT included in the current case

Directional Factors: 1.

Loads Applied

Assign Number	Vehicle Class	Scale Factor	Min Loaded Paths	Max Loaded Paths	Paths Loaded
	HL93	1.	2	2	
1	HL93	1.	2	2	All

MultiPath Scale Factors

Number of Paths Loaded	Reduction Scale Factor
	1
1	1.
2	1.

Paths Loaded for Assignment 1

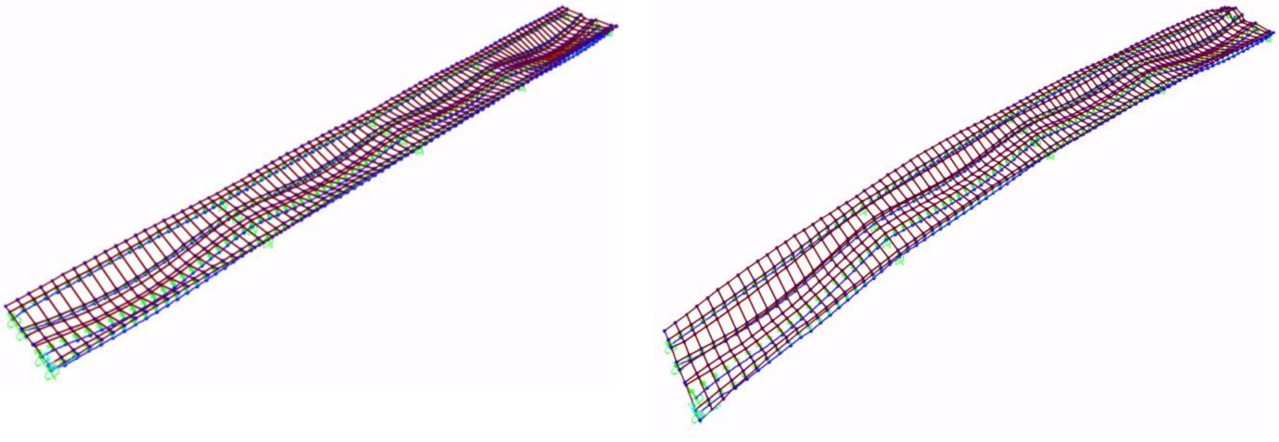
List of Path Definitions:

Selected Path Definitions: BOTTOMWHEEL
TOPWHEEL

Mass Source: MSSSRC1

Figure 7: Moving Load Case Parameters in SAP2000

Using these parameters, the bridge structures were analyzed. With the moving load case, the envelopes of the deformed shapes and underlying girder moments were outputted by SAP2000. The envelope of the deformed shape of the straight bridge and of the curved bridge are presented in Figure 8.



(a)

(b)

Figure 8: Envelope of Deformed Shapes: (a) the Straight Bridge; and (b) Curved Bridge (Right) from AASHTO HL-93 Design Truck

4 Results and Discussions

Using the data outputted by SAP2000 from each of the bridge models, analysis was conducted to calculate the LLDFs and to determine the impact of curvature on these factors.

4.1 Moment Envelopes

To visualize the difference in the structural responses of the curved and non-curved bridges, the envelope of the bending moment under the moving load was plotted for each girder of each bridge. The plot of the bending moment envelopes for the straight bridge is presented in Figure 9 (Note: 1 ft = 0.3048 m; 1 lb = 4.45 N). The naming convention of each girder is presented in Figure 4.

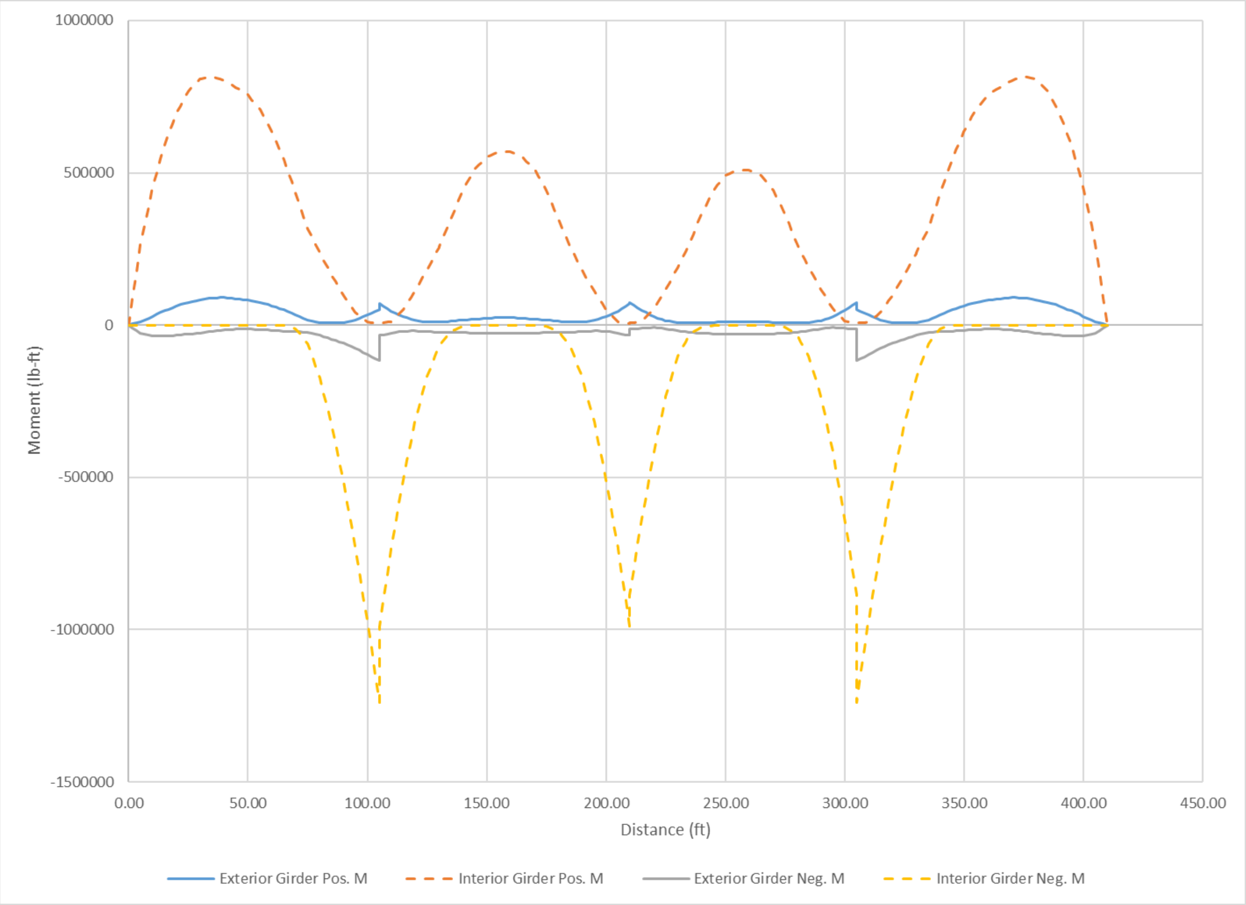


Figure 9: Moment Envelope for Underlying Girders of Straight Bridge

The plot of the bending moment envelopes for the curved bridge is presented in Figure 10.

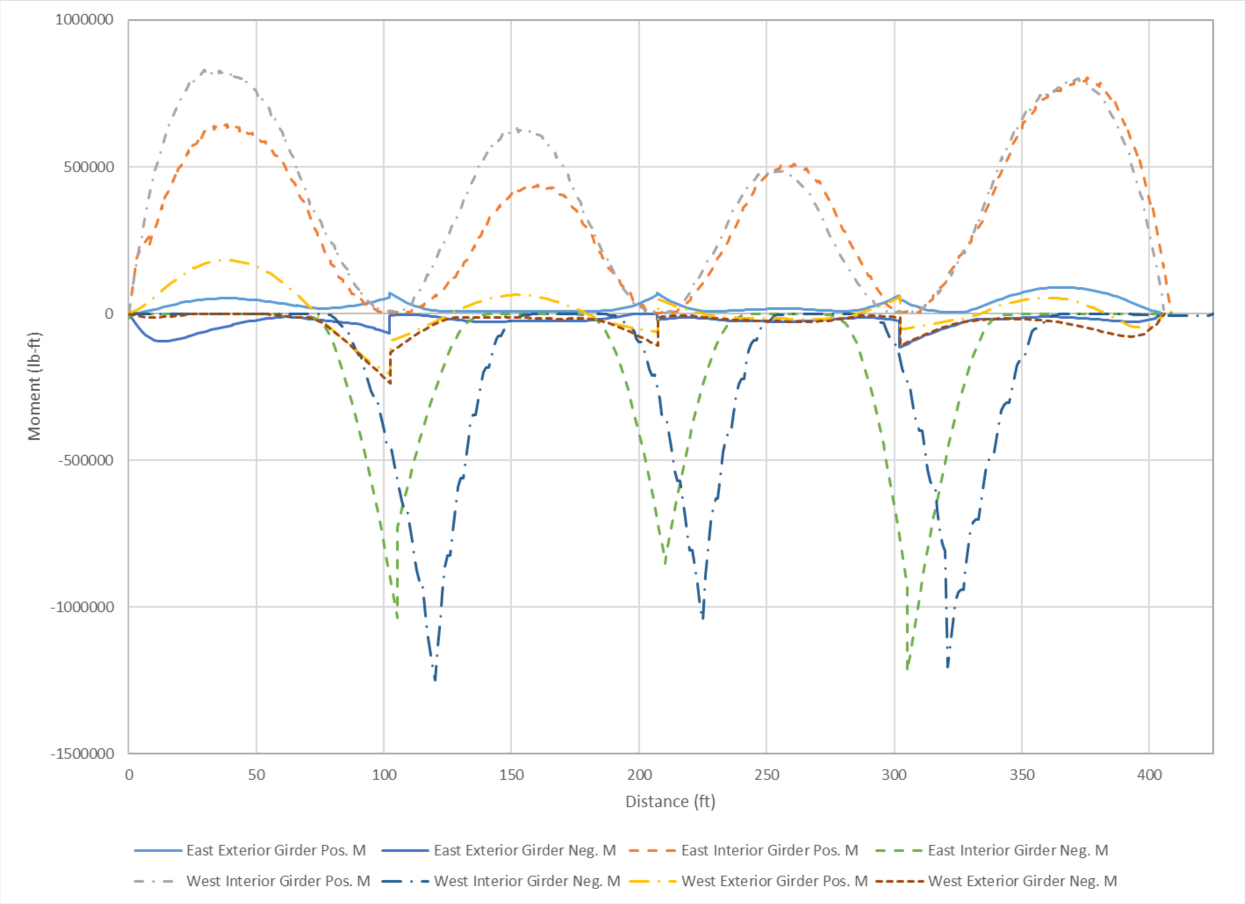


Figure 10: Bending Moment Envelope for Underlying Girders of Curved Bridge

As can be seen in Figure 9 and Figure 10, there is variation in the distribution of moment between the girders. For the straight bridge, the live load is evenly distributed between the two interior girders and two exterior girders. When curvature is introduced, the live load is unevenly distributed between the four girders. The variation in load distribution is further discussed in Section 5.

4.2 Calculation of LLDFs

The LLDFs for each girder for each bridge were calculated using the bending moment data outputted from SAP2000. LLDFs were calculated for each girder at each of the four spans for the positive moment and negative moment using the relationship

[1] $$LLDF = \frac{M_{max}}{\Sigma M_{max}}$$

where *LLDF* is the live load distribution factor, M_{max} is the maximum positive or negative moment, and ΣM_{max} is the sum of the maximum moments on each of the girders (Zaki, 2016). The LLDFs for the straight bridge girders are presented in Table 1.

Table 1: LLDFs for Bending Moments in Girders of the Straight Bridge

	Span 1		**Span 2**		**Span 3**		**Span 4**	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.
East Ext. Girder	0.0332	0.0346	0.0622	0.0146	0.0701	0.0107	0.0512	0.0555
East Int. Girder	0.3875	0.2758	0.3949	0.5374	0.5168	0.3301	0.4604	0.5892
West Int. Girder	0.4982	0.4577	0.5682	0.5879	0.4956	0.4049	0.4582	0.5811
West Ext. Girder	0.1084	0.0870	0.0692	0.0690	0.0718	0.0108	0.0471	0.0523

The LLDFs for the curved bridge girders are presented in Table 2.

Table 2: LLDFs for Bending Moments in Girders of the Curved Bridge

	Span 1		**Span 2**		**Span 3**		**Span 4**	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.
East Ext. Girder	0.0323	0.0404	0.0568	0.0120	0.0607	0.0141	0.0503	0.0434
East Int. Girder	0.3771	0.3225	0.3607	0.4445	0.4476	0.4362	0.4527	0.4610
West Int. Girder	0.4849	0.5352	0.5190	0.4862	0.4292	0.5351	0.4505	0.4546
West Ext. Girder	0.1055	0.1017	0.0632	0.0570	0.0622	0.0144	0.0464	0.0409

As can be seen in Table 1 and Table 2, there is variation in the LLDFs for the bending moments in girders of the straight bridge and those of the curved bridge. The difference of each distribution factor was calculated using the percent difference equation

[2] $$\%\,Diff = \frac{LLDF_1 - LLDF_2}{(LLDF_1 + LLDF_2)/2}$$

and is presented in Table 3. A positive difference indicates that the LLDF for the curved bridge was larger than that of the straight bridge. A negative difference indicates that the LLDF for the straight bridge was larger than that of the curved bridge.

Table 3: Percent Difference of LLDFs for Bending Moments in Girders

	Span 1		**Span 2**		**Span 3**		**Span 4**	
	Pos. M	Neg. M	Pos. M	Neg. M	Pos. M	Neg. M	Pos. M	Neg. M
East Ext. Girder	42.34%	167.38%	21.01%	9.44%	-4.11%	41.76%	39.93%	19.19%
East Int. Girder	17.65%	-151.89%	17.46%	9.06%	-1.35%	9.20%	-6.43%	-3.01%
West Int. Girder	-7.42%	-169.55%	-18.82%	0.09%	2.85%	-11.20%	-5.94%	-1.61%
West Ext. Girder	-71.80%	126.99%	10.37%	-124.49%	-6.52%	40.00%	47.68%	25.05%

5 Conclusions

Two bridges were modeled using the finite element modeling and analysis procedures presented in Section 3. One curved bridge and one straight bridge with similar properties were modeled to analyze the effect of curvature on LLDFs. Per the percent differences of the LLDFs presented in Table 3, the LLDFs of the two bridges vary from a negligible difference (0.09%) to a difference of 169.55%.

The difference in load distribution is visualized in the plots of the bending moment envelope of each bridge presented in Figure 9 and Figure 10. As shown in Figure 9, the load is distributed evenly in both exterior girders and both interior girders in the straight bridge. As shown in Figure 10, the distribution of load is not symmetrical in the curved bridge.

Per pervious research done investigating the effects of curvature on concrete box girder bridges, it was observed that an increase in the degree of curvature of a bridge directly correlates with an increase in LLDFs in girders (Zaki, 2016). This observation did not hold true in the case of this study, as can been in Table 3, since approximately half of the LLDFs calculated for the curved bridge were smaller than those for the straight bridge. It is believed that the results of this study do not correlate with the conclusions of the previous research due to the difference in bridge geometry that was analyzed. The curved bridges that were analyzed by Zaki were supported by curved girders as opposed to the straight girders that support the bridges analyzed in this study. Due to the eccentric geometry of the straight girders, additional flange forces, web forces, and tensile forces that were not present in the bridges analyzed by Zaki likely impacted the distribution of loads for the bridges analyzed in this study such that the LLDFs did not increase as a function of curvature (Lewis, 2016).

Further investigation into the curvature effects on LLDFs for horizontally curved bridges supported by straight concrete girders is recommended. Using the finite element modeling and statistical analysis procedures presented in this study, multiple bridges of varying curvature may be analyzed to approximate the difference in LLDFs as a function of increasing degree of curvature. Additionally, parametric studies are recommended to determine how changes in girder orientation, span length, and the number of girders impact the LLDFs of these bridges.

Acknowledgements

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